

CP-odd effective gluonic Lagrangian in the Kobayashi-Maskawa model

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Abstract

Schwinger operator method is applied for studying CP-odd pure gluonic effective Lagrangian in the Standard Model at three-loop level. The induced θ -term vanishes by the same reasons as EDMs of quark and W-boson to two-loop approximation. A simple way is found to demonstrate these cancellations. All other terms of the effective Lagrangian acquire non-vanishing contributions. The effective operator of dimension six, Weinberg operator, is calculated explicitly. The corresponding contribution to the EDM of neutron is much smaller than that comes from large distances.

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1 Introduction

The Kobayashi-Maskawa (KM) model looks now as the most natural description of CP-violation. It describes properly CP-odd phenomena in the decays of neutral K -mesons and predicts extremely tiny CP-odd effects in the flavour-conserving processes. Though its predictions for the electric dipole moments (EDM) of elementary particles are far beyond the present experimental facilities, the corresponding theoretical investigations are of certain methodological interest.

The subject of this work is the calculation of the CP-odd effective gluonic Lagrangian which appears in the Standard Model as a result of integration over quark and W-boson modes. This Lagrangian can be naturally expanded in the series of gluon field operators of increasing dimension:

$$S_{eff} = \int d^4x \mathcal{L}_{eff}(x) = \int d^4x \left(c_1 g^2 G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + c_2 g^3 f^{abc} \tilde{G}_{\alpha\beta}^a G_{\beta\mu}^b G_{\mu\alpha}^c + \dots \right), \quad (1)$$

where g is the chromoelectric charge, $\tilde{G}_{\alpha\beta}^a = 1/2 G_{\mu\nu}^a \epsilon_{\mu\nu\alpha\beta}$. It is assumed that the characteristic loop momenta are much larger than the inverse scale on which field fluctuations occur. The first term in (1) represents the induced θ -term, perturbative contribution to the total θ -term of the theory. The next operator of dimension 6 was introduced originally by Weinberg [1]. In different classes of models violating CP-symmetry this operator may give an important contribution to the neutron electric dipole moment [2, 3].

The violation of CP-symmetry in Standard Model originates from the complexity of the KM matrix. To lowest, quadratic order in the weak interaction all CP-odd flavour-conserving amplitudes turn to zero trivially. The point is that in this approximation those amplitudes depend only on the moduli squared of elements of the KM matrix, so the result cannot contain the CP-violating phase.

CP-odd objects may arise in the Standard Model in the fourth order in semi-week constant. However, the cancellation of EDMs of a quark and W-boson in this approximation is firmly established now [4, 5]. The only known non-vanishing formfactor to this approximation is the magnetic quadrupole moment of W-boson [6]. The finite EDMs can be obtained only after hard gluon radiative corrections are taken into account. We shall prove that in the absence of QCD radiative corrections the same mechanism leads to the cancellation of induced θ -term. In contrary to the recent claim that the Weinberg operator is zero to this approximation [7], we find that all operators of $\dim \geq 6$ acquire non-vanishing values.

2 Schwinger operator method for calculating CP-odd Lagrangian

We are going over now to the direct calculation of a few first terms of CP-odd effective gluonic Lagrangian in the Standard Model to three-loop approximation. The general structure of the diagrams which could contribute to the effect in that approximation is shown on Fig.1, where the solid line represents a quark loop, waved lines - W-bosons.

The CP-odd part of the loop flavour structure reads as:

$$2i\tilde{\delta}[d(c(b-s)t - t(b-s)c + t(b-s)u - u(b-s)t + u(b-s)c - c(b-s)u) \\ + s(c(d-b)t - t(b-s)c + t(d-b)u - u(d-b)t + u(d-b)c - c(d-b)u) \\ + b(c(s-d)t - t(s-d)c + t(s-d)u - u(s-d)t + u(s-d)c - c(s-d)u)] \quad (2)$$

For the KM matrix we use the standard parameterization of Ref.[8] where the CP-odd invariant is

$$\tilde{\delta} = \sin \delta c_1 c_2 c_3 s_1^2 s_2 s_3. \quad (3)$$

The letters u, d, s, c, b, t denote here the Green's functions of the corresponding quarks. Each product of four quark propagators allows for cyclic permutations of the kind

$$udcs = dcsu = csud = sudc.$$

Further considerations are based on the operator Schwinger method [9] successfully extended on the QCD case by Novicov, Shifman, Vainshtein and Zhakharov [10]. It allows one to minimize the set of calculations in introducing the operator \hat{P} :

$$\langle x|\hat{P}|y\rangle = \langle x|i\hat{D}|y\rangle = \gamma_\mu(i\frac{\partial}{\partial x_\mu} + g\frac{\lambda^c}{2}A_\mu^c(x))\delta^4(x-y), \quad (4)$$

where $A_\mu^c(x)$ is the external gluonic field. Then the quark propagator taken in the background gluonic field reads as:

$$\langle vac|Tq^a(x)\bar{q}^b(y)|vac\rangle = \langle x,a|i(\hat{P}-m)^{-1}|y,b\rangle = \langle x,a|(\hat{P}-m)\frac{i}{P^2 + ig/2(G\sigma) - m^2}|y,b\rangle. \quad (5)$$

The external field strength appears as a result of commutation of two P :

$$[P_\mu, P_\nu] = iG_{\mu\nu}^a \frac{\lambda^c}{2} \equiv igG_{\mu\nu} \quad (6)$$

We assume also that the field has no source and satisfies classical equations of motion²:

$$D_\mu G_{\mu\nu} = 0; \quad D_\mu \tilde{G}_{\mu\nu} = 0. \quad (7)$$

The general outline of our calculation is following. Using the specific property of the flavour summation (2), namely the antisymmetrization in the masses of opposite fermionic lines, we rewrite the general expression for the CP-odd amplitude in KM model via some commutators of functions depending on P . It gives us some powers of external field and its derivatives in the nominator. If the explicit dimension of this part of expression is equal to the dimension of operator of interest, we can forget about further non-commutativity of P s in other parts and put $A = 0$.

The generic formula for the effective action up to some renormalization terms which will be discussed later looks as

$$S_{eff} = -i \sum_{flavour} \text{Tr} \left[\frac{\hat{P} + m_1}{\hat{P}\hat{P} - m_1^2} \hat{\Gamma}(m_2^2) \frac{1 + \gamma_5}{2} \frac{\hat{P} + m_3}{\hat{P}\hat{P} - m_3^2} \hat{\Gamma}(m_4^2) \frac{1 + \gamma_5}{2} \right], \quad (8)$$

²As far as we are interested in pure gluonic operators we can omit quark currents in the r.h.s. of (7)

where the sum over quark's masses m_1, m_3 and m_2, m_4 should be performed according (2). The Tr operation in this equation means the trace in colour, Lorentz and coordinate spaces:

$$\text{Tr}[\dots] \equiv \text{Tr}_{L+C} \int d^4x \langle x | \dots | x \rangle, \quad (9)$$

$\hat{\Gamma}$ denotes the mass operator of a quark in the external field:

$$\hat{\Gamma}(m_i^2) = \frac{g_w^2}{2} \int \frac{d^4q}{(2\pi)^4} \gamma_\mu \frac{1 + \gamma_5}{2} \frac{\hat{q} - \hat{P}}{(q - P)^2 + ig/2(G\sigma) - m_i^2} \gamma_\nu \frac{1 + \gamma_5}{2} \frac{g_{\mu\nu} - q_\mu q_\nu / M^2}{q^2 - M^2}, \quad (10)$$

where M is the mass of the W-boson, g_w - semiweak charge. This mass operator $\hat{\Gamma}$ allows for the expansion in series of external field operators of increasing dimension with some invariant functions depending on P^2 as coefficients. It will be shown later that for the calculation of the Weinberg operator it is sufficient to keep in this expansion only three first terms of *explicit* dimension 0, 2 and 3.:

$$\hat{\Gamma}(m_i^2) = \left(\frac{1}{2} \{ \hat{P}, F_i(P^2) \} + \frac{ig}{4} \{ H_i(P^2), \{ \hat{P}, G\sigma \} \} + gJ_i(P^2) D_\alpha G_{\mu\nu} P_\alpha P_\mu \gamma_\nu + \dots \right) \frac{1 + \gamma_5}{2} \quad (11)$$

Here $\{\dots\}$ denotes anticommutator. Some comments should be added at this point. As far as we do not fix the concrete view of invariant functions, two last terms in (11) could be presented in the other form. The term of dimension 2, for instance, could be written as $\{ \hat{P}, \{ H_i(P^2), G\sigma \} \}$. The difference between these two forms, however, is an operator of dimension 4 and it can be absorbed to the next term of this expansion. From the same reasons we do not care about the antisymmetrization in the last term in (11). The only thing which should be checked is the absence of operators $\{ H_i(P^2), \{ \hat{P}, \tilde{G}\sigma \} \}$ and $D_\alpha \tilde{G}_{\mu\nu} P_\alpha P_\mu \gamma_\nu$. This could be made from the expression (10) or in the framework of the usual perturbative expansion.

The V - A structure of $\hat{\Gamma}$ cancels m_1 and m_3 in nominators of (8). The only problem arises with renormalization terms which violate pure left-handedness. Now we shall argue that renormalization never contributes to the CP-odd effective Lagrangian to this approximation.

In the absence of external field the first term in (11) reduces to the usual unrenormalized mass operator in the V-A theory:

$$\frac{1}{2} \{ \hat{P}, F(P^2) \} \frac{1 + \gamma_5}{2} \Big|_{A=0} = \hat{p} \frac{1 + \gamma_5}{2} f(p^2). \quad (12)$$

The renormalization with respect to quark 1 from the left and quark 3 from the right introduces into the mass operator the dependence of external masses [4, 5]:

$$\hat{p} \frac{1 + \gamma_5}{2} \tilde{f}(p^2) - f_{13} \left[\hat{p} \frac{1 - \gamma_5}{2} - m_1 \frac{1 - \gamma_5}{2} - m_3 \frac{1 + \gamma_5}{2} \right], \quad (13)$$

where f_{13} and \tilde{f} are expressed via the function f and masses m_1, m_3 as follows:

$$\tilde{f}(p^2) = f(p^2) - \frac{m_1^2 f_1 - m_3^2 f_3}{m_1^2 - m_3^2}, \quad f_{13} = \frac{m_1 m_3 (f_1 - f_3)}{m_1^2 - m_3^2}; \quad f_i = f(p^2 = m_i^2), \quad i = 1, 3. \quad (14)$$

It is clear how to adopt this scheme for the formalism of the external field. Now the first term of (11) should be written in the following form:

$$\left(\frac{1}{2}\{\hat{P}, F(P^2)\} - \hat{P} \frac{m_1^2 f_1 - m_3^2 f_3}{m_1^2 - m_3^2}\right) \frac{1 + \gamma_5}{2} - \frac{f_{13}}{2} [\hat{P}(1 - \gamma_5) - m_1(1 - \gamma_5) - m_3(1 + \gamma_5)], \quad (15)$$

where constants f_1 , f_3 and f_{13} are determined in (14). The zeroth-order term of the perturbative expansion of this expression in A reproduces (13); the first-order term corresponds to the vertex part renormalized in complying with the Ward identity. Other terms appear to be free of renormalization.

Let us start our consideration of renormalization effects from the counterterms of the "wrong" handedness, proportional to f_{13} . From flavour structure (2) of the fermionic loop it follows, in particular, that any amplitude should be antisymmetrized in the masses m_1 and m_3 of the opposite quark's lines. The cyclic permutation allowed under the trace symbol simultaneously leads to the antisymmetry with respect to interchange of m_2 and m_4 . The last property automatically means the vanishing of the amplitude proportional to $f_{13}(m_2)f_{13}(m_4)$. Thus, one of mass operators at least possesses V - A structure. Then the contribution of counterterms, proportional to f_{13} can be easily evaluated to the form:

$$\begin{aligned} \text{Tr} \left[\frac{\hat{P} + m_1}{\hat{P}\hat{P} - m_1^2} \frac{f_{13}}{2} (\hat{P}(1 - \gamma_5) - m_1(1 - \gamma_5) - m_3(1 + \gamma_5)) \frac{\hat{P} + m_3}{\hat{P}\hat{P} - m_3^2} \hat{\Gamma}(m_4^2) \frac{1 + \gamma_5}{2} \right] = \\ -f_{13}m_1m_3 \text{Tr} \left[\frac{\hat{P}}{(p^2 + i/2G\sigma - m_1^2)(p^2 + i/2G\sigma - m_3^2)} \hat{\Gamma}(m_4^2) \frac{1 + \gamma_5}{2} \right]. \quad (16) \end{aligned}$$

This expression is explicitly symmetric under the permutation $m_1 \leftrightarrow m_3$ and drops out from the answer. Therefore, only V - A structures left in both operators $\hat{\Gamma}$ and our master formula reduces to

$$S_{eff} = -i \sum_{flavour} \text{Tr} \left[\frac{\hat{P}}{\hat{P}\hat{P} - m_1^2} \hat{\Gamma}(m_2^2) \frac{\hat{P}}{\hat{P}\hat{P} - m_3^2} \hat{\Gamma}(m_4^2) \frac{1 + \gamma_5}{2} \right]. \quad (17)$$

The procedure of the antisymmetrization in $m_1 \leftrightarrow m_3$ implied at this point significantly simplifies the set of further calculations. Indeed, the m_1, m_3 -dependent part of the amplitude can be easily transformed as follows:

$$\begin{aligned} \frac{\hat{P}}{\hat{P}\hat{P} - m_1^2} \hat{\Gamma} \frac{\hat{P}}{\hat{P}\hat{P} - m_3^2} - (m_1 \leftrightarrow m_3) = \frac{\hat{P}}{\hat{P}\hat{P} - m_1^2} \hat{\Gamma} \frac{\hat{P}\hat{P} - m_1^2}{\hat{P}\hat{P} - m_1^2} \frac{\hat{P}}{\hat{P}\hat{P} - m_3^2} - (m_1 \leftrightarrow m_3) = \\ \frac{\hat{P}}{\hat{P}\hat{P} - m_1^2} [\hat{\Gamma}, \hat{P}\hat{P}] \frac{\hat{P}}{(\hat{P}\hat{P} - m_1^2)(\hat{P}\hat{P} - m_3^2)} - (m_1 \leftrightarrow m_3) = (m_1^2 - m_3^2) \hat{P} \hat{S}_{13} [\hat{\Gamma}, \hat{P}\hat{P}] \hat{S}_{13} \hat{P}, \quad (18) \end{aligned}$$

where we have introduced the operator $\hat{S}_{13} = (\hat{P}\hat{P} - m_1^2)^{-1}(\hat{P}\hat{P} - m_3^2)^{-1}$. It is easy to see that the antisymmetrization in $m_1 \leftrightarrow m_3$ performed in (18) leads to the antisymmetry of the amplitude with respect to interchange of indices 2 and 4. Indeed, using the cyclic permutation we get:

$$(m_1^2 - m_3^2) \text{Tr} \left(\hat{P} \hat{S}_{13} [\hat{\Gamma}_2, \hat{P}\hat{P}] \hat{S}_{13} \hat{P} \hat{\Gamma}_4 \frac{1 + \gamma_5}{2} \right) = -(m_1^2 - m_3^2) \text{Tr} \left(\hat{P} \hat{S}_{13} [\hat{\Gamma}_4, \hat{P}\hat{P}] \hat{S}_{13} \hat{P} \hat{\Gamma}_2 \frac{1 + \gamma_5}{2} \right) \quad (19)$$

The calculation of the commutator in the expression (18) is quite straightforward. For instance, this commutator with the rest of the first term of the mass operator can be calculated in the following manner:

$$\begin{aligned} & \left[\left(\frac{1}{2} \{ \hat{P}, F(P^2) \} - \hat{P} \frac{m_1^2 f_1 - m_3^2 f_3}{m_1^2 - m_3^2} \right), \hat{P} \hat{P} \right] \frac{1 + \gamma_5}{2} = \frac{1}{2} \{ \hat{P}, [F(P^2), P^2 - \frac{ig}{2} G\sigma] \} \frac{1 + \gamma_5}{2} \\ & = \frac{ig}{2} (\{ P_\alpha \{ F' \{ P_\mu, D_\mu \tilde{G}_{\alpha\beta} \} \} \} - \frac{ig}{4} \{ F' \{ P_\mu, D_\alpha D_\mu G_{\alpha\beta} \} \}) \gamma_\beta \frac{1 + \gamma_5}{2} + \mathcal{O}(\dim \geq 5). \end{aligned} \quad (20)$$

This formula is obtained using the relation from the general operator calculus:

$$2[F(A), B] = \{F'(A), [A, B]\} - \frac{1}{2!} [F''(A), [A, [A, B]]] + \frac{1}{3!} \{F'''(A), [A, [A, [A, B]]]\} - + \dots \quad (21)$$

The result of the commutation (20) brings some important consequences:

First, it cancels the rest of renormalization counterterms. This completes the prove of the statement that the renormalization never contributes to the CP-odd flavor diagonal amplitudes to the fourth order in semi-weak constant.

Second, the minimal explicit dimension of the operator $[\hat{\Gamma}, \hat{P}\hat{P}]$ is 3 and it puts the limit on the number of terms in $\hat{\Gamma}$ that we have to take into account. For the calculation of the Weinberg operator we can restrict our considerations on the operators of explicit dimension 3 or less and it justifies, in particular, the choice of $\hat{\Gamma}$ in the form (11).

Third, we have found the shortest way to demonstrate the absence of EDMs of the quark and W-boson to this approximation. There is no doubt that all considerations presented above can be extended on the case of the external electromagnetic field.

Now we are in the right position for the calculation of the Weinberg operator in KM model. It is convenient to classify all contributions by the combination of invariant functions from the expansion of $\hat{\Gamma}$'s. There are five of them: J-H, J-F, H-H, H-F, F-F. For the induced θ -term the only possible contribution may arise from F-F.

The simplest cases are J-H and F-H because they already possess $\dim=6$. Therefore, the further non-commutativity may only influence on the effective operators of higher dimension. Thus, it is reasonable to make the substitutions:

$$P \longrightarrow p; \quad \hat{S}_{13} \longrightarrow \frac{1}{(p^2 - m_1^2)(p^2 - m_3^2)} \quad (22)$$

Then the trace over spacial variables is easily computable:

$$\begin{aligned} S_{eff} &= 4g^2 \int d^4x \text{Tr}_C [(D_\mu \tilde{G}_{\alpha\beta})(D_\nu G_{\lambda\beta})] \times \\ & \sum_{flavour} \int \frac{d^4p}{(2\pi)^4} \frac{(m_1^2 - m_3^2) p_\alpha p_\mu p_\nu p_\lambda p^2 [J_4(F'_2 - H_2) - J_2(F'_4 - H_4)]}{(p^2 - m_1^2)^2 (p^2 - m_3^2)^2}, \end{aligned} \quad (23)$$

The trivial average over the direction of p produces three different field operators of the dimension 6. By virtue of equations of motion for the external field (7) they can be

transformed to the standard form of Weinberg operator. Finally, we get J-H and J-F contributions to the Weinberg operator in the form:

$$c_2 = i \sum_{flavour} \int \frac{d^4 p}{(2\pi)^4} \frac{(m_1^2 - m_3^2) p^6 [J_4(F'_2 - H_2) - J_2(F'_4 - H_4)]}{(p^2 - m_1^2)^2 (p^2 - m_3^2)^2} \quad (24)$$

The analysis of the H-H contribution is quite transparent. The corresponding amplitude contains field operators of dimension 5 and 6. It means that we can neglect the non-commutativity of H_2 , H_4 with operators \hat{P} and \hat{S}_{13} because it would bring an additional dimension 2. Then the amplitude of interest could be transformed to the form:

$$\text{Tr}(\{H_2, [\hat{O}, \hat{P}\hat{P}]\}\{H_4, \hat{O}\}) = \frac{1}{2} \text{Tr}(\{H_2, [\hat{O}, \hat{P}\hat{P}]\}\{H_4, \hat{O}\}) - \{H_4, [\hat{O}, \hat{P}\hat{P}]\}\{H_2, \hat{O}\}, \quad (25)$$

where $\hat{O} = ig/4\{\hat{P}, G\sigma\}$. It is the matter of simple exercise to check that the expression (25) vanishes identically.

We left with the F-F and H-F groups of contributions which minimal dimension is 3. The straightforward calculation is quite tedious because we have to take into account $G\sigma$ -dependence of \hat{S}_{13} , the non-commutativity of different P , etc. However, we have found a simple argument to show these groups do not contribute to the Weinberg operator at all. In the amplitude of interest (F-F case)

$$\text{Tr}(\hat{P}\hat{S}_{13}[\{\hat{P}, F_2(P^2)\}, \hat{P}\hat{P}]\hat{S}_{13}\hat{P}\{\hat{P}, F_4(P^2)\}), \quad (26)$$

we perform the formal expansion in $ig/2G\sigma$ of $F_i(P^2) = F_i(P^2 + ig/2G\sigma - ig/2G\sigma)$ around the "point" $P^2 + ig/2G\sigma = \hat{P}\hat{P}$. It is clear that only zeroth and first order terms of that expansion could be taken into account when calculating Weinberg operator. This expansion can be performed using another formula of the general operator calculus. Up to terms linear in B , function $F(A + B)$ could be expanded in the following manner:

$$2F(A + B) = 2F(A) + \{F'(A), B\} + \frac{1}{2!}[F''(A), [A, B]] + \dots \quad (27)$$

In our case this expansion takes the form:

$$F(\hat{P}\hat{P} - ig/2G\sigma) = F(\hat{P}\hat{P}) - \frac{1}{2}\{F'(\hat{P}\hat{P}), ig/2G\sigma\} + \frac{1}{4}[F''(\hat{P}\hat{P}), [\hat{P}\hat{P}, -ig/2G\sigma]] + \dots \quad (28)$$

Only second term of this expansion is relevant in our consideration. Indeed, the third term and other denoted here as ... have the dimension higher than 3 and will not contribute to the Weinberg operator. First terms of the expansion of F_2 and F_4 drop out because they give a vanishing commutator with $\hat{P}\hat{P}$. So, we left with the second terms only but their contribution literally coincides with H-H case. Thus, the vanishing of H-F and F-F groups of contributions follows from the simple substitution $H_i \longrightarrow H_i - F'_i$ in equation (25). Another basis of invariant functions depending on $\hat{P}\hat{P}$ in the expansion of $\hat{\Gamma}$

$$\hat{\Gamma}_i = (\{\hat{P}, \tilde{F}_i(\hat{P}^2)\}) + \frac{ig}{4}\{\tilde{H}_i(\hat{P}^2), \{\hat{P}, G\sigma\}\} + g\tilde{J}_i(\hat{P}^2)D_\alpha G_{\mu\nu}P_\alpha P_\mu \gamma_\nu + \dots \frac{1 + \gamma_5}{2}. \quad (29)$$

would simplify our analysis. It reduces the number of possible combinations for the calculation of the Weinberg operator to $\tilde{H} - \tilde{H}$ and $\tilde{H} - \tilde{J}$.

We have shown that the KM-model does not induce θ -term to three-loop approximation. To the same accuracy the Weinberg operator acquires nonvanishing contributions of the form (24).

3 Weinberg operator in the KM model

After convincing ourselves in the absence of the exact cancellation of the Weinberg operator in three-loop approximation, we are going to find its value. It is natural to consider all quark masses but m_t small as compared to the W-boson one M . Together with the quark mass hierarchy it allows one to simplify the calculations considerably, restricting to those contributions to the operator of interest which are of lowest order in the light quark masses. Besides, it is also natural to single out the contributions with logarithms of large mass ratios, e.g., $\log(m_t/m_c)$, $\log(m_b/m_s)$, $\log(M/m_b)$ etc.

All diagrams can be split into two types, depending on which quarks, U (u, c, t) or D (d, s, b), flow inside the mass operators. It is convenient to sum first of all over the flavours of the quarks masses of which were denoted up to now as m_1 and m_3 . For the two types mentioned we get respectively:

$$\sum \frac{(m_1^2 - m_3^2)}{(p^2 - m_1^2)^2(p^2 - m_3^2)^2} \longrightarrow \frac{-m_b^4 m_s^2}{p^4(p^2 - m_b^2)^2(p^2 - m_s^2)^2} - \frac{-m_t^4 m_c^2}{p^4(p^2 - m_t^2)^2(p^2 - m_c^2)^2}. \quad (30)$$

In expression (30) we put $m_u = m_d = 0$. We can determine now the characteristic momenta p . When quarks are arranged according to the first line of formula (30), integral (24) is infrared divergent if one neglects the masses m_s and m_b in the denominator. It means that the typical loop momenta contributing to the effect are $p \sim m_b$ and it cancels two powers of m_b in the nominator of (30). In the opposite case when D -quarks are inside the mass operators, the typical momenta range is large: $p \sim M$.

The main problem arising at this point is the calculation of invariant functions F , H and J . This could be done by means of usual perturbative expansion of formulae (10) and (11). Using the fixed point gauge

$$(z - x)_\mu A_\mu(z) = 0 \quad (31)$$

$$A_\mu(z) = \frac{1}{2}(z - x)_\nu G_{\nu\mu}(x) + \frac{1}{3}(z - x)_\alpha (z - x)_\nu D_\alpha G_{\nu\mu}(x) + \dots$$

after straightforward calculations we get the following set of equations:

$$F(p^2)\hat{p} = \tilde{F}(p^2)\hat{p} = \frac{g_w^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{-2\hat{q} - \frac{1}{M^2}(\hat{p}\hat{q}\hat{p} + q^2\hat{q} - 2q^2\hat{p})}{(q^2 - m^2)[(p - q)^2 - M^2]} \quad (32)$$

$$H(p^2)\tilde{G}_{\mu\nu}p_\mu\gamma_\nu = (\tilde{H} + \tilde{F})(p^2)\tilde{G}_{\mu\nu}p_\mu\gamma_\nu = \frac{g_w^2}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{(\gamma_\mu q_\nu - \frac{2}{M^2}\hat{q}p_\mu q_\nu)\tilde{G}_{\mu\nu}}{(q^2 - m^2)^2[(p - q)^2 - M^2]} \quad (33)$$

$$J(p^2)D_\alpha G_{\nu\mu} p_\nu p_\alpha \gamma_\mu = \frac{g_w^2}{3} \int \frac{d^4 q}{(2\pi)^4} \frac{(\gamma_\mu - \frac{2}{M^2} \hat{q} p_\mu) q_\nu q_\alpha D_\alpha G_{\mu\nu}}{(q^2 - m^2)^3 [(p - q)^2 - M^2]} + \frac{F''}{3} D_\alpha G_{\nu\mu} p_\nu p_\alpha \gamma_\mu \quad (34)$$

The cubic divergence of integral in the expression for F is irrelevant for us because the combination $F'(m_i^2) - F'(m_j^2)$ presenting in our equations is obviously finite.

Before taking the integrals in (32) - (34) it is reasonable to determine the light quark mass dependence of F' , F'' , H and J and sum over flavours left. Clearly, the summation $\sum [J_4(F'_2 - H_2) - J_2(F'_4 - H_4)]$ brings an additional factor m_c^2 for U -quarks, so the total degree of suppression for this type of diagrams is $\mathcal{O}(m_b^2 m_c^2 m_s^2)$. For the D -type of quark's arrangement it is essential that functions F' and F'' could be expanded as follows:

$$F'(p^2, m^2) = F'(p^2, m^2 = 0) + m^2 \frac{dF'}{d(m^2)} \Big|_{m=0} + \dots, \quad (35)$$

whereas H and J contain pieces proportional to $m^2 \log m^2$ (see the Ref.[6] for the details). In the sum over D -flavours these logarithms prevent the cancellation of terms $\sim m_b^2 m_s^2$ which means that the group of diagrams with D -quarks inside mass operators contribute to the Weinberg operator at the same order $\mathcal{O}(m_b^2 m_c^2 m_s^2)$. Moreover, these logarithmic factors enhance the contribution came from D -type of quark arrangement in comparison with that from U -type. Up to the last integral over p^2 the corresponding contribution to the Weinberg operator to double logarithmic accuracy reads as:

$$c_2 = -\frac{i\tilde{\delta}}{3} \left(\frac{g_w^2}{32} \right)^2 \frac{m_b^2 m_c^2 m_s^2}{M^4} \log(m_b^2/m_s^2) \int \frac{d^4 p}{(2\pi)^4} \frac{(p^2 + M^2) m_t^4}{(p^2 - M^2)^5 (p^2 - m_t^2)^2} \log \frac{|p^2 - M^2|}{m_b^2}. \quad (36)$$

Performing trivial integration we get the final formula for the Weinberg operator in the Kobayashi-Maskawa model:

$$\frac{1}{1536\pi^6} \tilde{\delta} G_F^2 \frac{m_b^2 m_c^2 m_s^2}{M^4} \log(m_b^2/m_s^2) \log(M^2/m_b^2) I(m_t^2/M^2), \quad (37)$$

where the function I is

$$I(x) = \frac{x^2}{(x-1)^4} \log x \left(3 + \frac{12}{x-1} + \frac{10}{(x-1)^2} \right) - \frac{x}{(x-1)^2} \left(3 + \frac{13}{x-1} + \frac{5}{(x-1)^2} + \frac{10}{(x-1)^3} \right). \quad (38)$$

The Fermi constant is introduced in (37) according the standard notation $G_F = \sqrt{2}g^2/(8M^2)$.

4 Discussion

We have shown that the Kobayashi-Maskawa model generates CP-odd effective gluonic Lagrangian to three-loop approximation starting from the operator of dimension 6. The expression for the Weinberg operator (37) parametrically coincides with the effective magnetic quadrupole moment of W -boson [6] appearing in this model in the same fourth order in semi-weak constant. The attempt to prove the exact cancellation of the Weinberg operator using the external field technic [7] seems to be incorrect. The author of this work believes that antisymmetrizations of the amplitude (8) in m_1 , m_3 and m_2 , m_4 should be imposed

independently and *both* procedures increase the effective dimension. In contrary, we have shown they are connected and the amplitude antisymmetrized in one pair is automatically antisymmetric in another one.

There is nothing surprising in the fact that the contribution to the electric dipole moment of neutron came from (37) is tremendously small. For $m_t \sim 2M$ we get

$$c_2 \simeq 10^{-27} (1 \text{ GeV})^{-2} \quad (39)$$

Using the result of the work [11] we can estimate the corresponding contribution to the NEDM as 10^{-41} e cm . It is far beyond both current experimental limit and theoretically predicted NEDM came from large distances [12] as well. The extreme smallness of (39) reflects not only usual parametric suppression of the effect but amazingly small numerical coefficient as well.

Finally, we would like to discuss a possible value of the Weinberg operator at four-loop level. One additional hard gluon loop brings a factor like $\alpha_s/(3\pi) \sim 10^{-2}$ but now the operator of interest appears in another order in light quark masses. As a result we could expect the corresponding coefficient c_2 being much larger than its three-loop value [2]. To obtain an estimation for the Weinberg operator we use the approach developed in the work [13]. Believing that all quark masses are much smaller than the mass of W -boson one can use four-fermion contact limit restricting on the contributions to the effective Lagrangian of order M^{-4} . The only possible structure of diagrams is presented on the Fig.2. (Dashed line here is the gluon propagator). The induced θ -term appears from this graph in the order $\mathcal{O}(\alpha_s G_F^2 m_s^2 m_c^2)$ [13]. The heaviest masses enters only under logarithms here. The estimation for the Weinberg operator quoted in [2], $\mathcal{O}(\alpha_s G_F^2 m_c^2)$, looks strange now because the ratio c_2/c_1 is of order $1/m_s^2$. If it is true it makes questionable the validity of the expansion (1) at the four-loop level. Performing the same analysis we came, however, to the another estimation of the effect. The typical expression for the corresponding amplitude before the last integral over gluon momentum k^2 looks as:

$$\alpha_s G_F^2 \int dk^2 \frac{m_c^2}{k^2} \log(m_t^2/k^2) \log(k^2/m_c^2) \frac{m_s^2}{k^2} \log(m_b^2/k^2), \quad (40)$$

where the infrared divergence should be cut off at the scale of m_c^2 . Correspondingly, the four-loop contribution to the Weinberg operator is of order $\mathcal{O}(\alpha_s G_F^2 m_s^2)$ which is two order of magnitude smaller than the estimation cited above.

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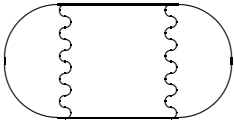


Fig. 1

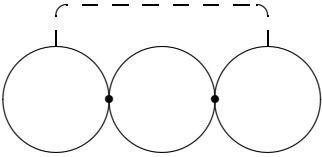


Fig. 2